On the sensitivity of the energy of vanishing flow towards mass asymmetry of colliding nuclei

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Abstract

We demonstrate the role of the mass asymmetry in the energy of vanishing flow by studying asymmetric reactions throughout the periodic table and over entire colliding geometry. Our results, which are almost independent of the system size and as well as of the colliding geometries indicate a sizable effect of the asymmetry of the reaction on the energy of vanishing flow.

Keywords: heavy-ion collisions, multifragmentation, quantum molecular dynamics (QMD) model, energy of vanishing flow, mass asymmetric reactions

1. Introduction

In the search of nuclear equation of state as well as of nuclear interactions and forces, collective flow has been found to be of immense importance [1, 2, 3, 4]. Among collective flow, transverse in-plane flow enjoys special status. During the last two decades, much emphasis has been put on the study of collective flow [1, 2, 3, 4]. Lots of experiments have been performed and number of theoretical attempts have also been employed to explain and understand these observations. As reported by [5] for the first time and later on by many others, collective flow is negative at low incident energies whereas it is positive at a reasonable higher incident energies. At a particular incident energy, however, a transition occurs. This transition energy is also known as energy of vanishing flow or balance energy. This energy of vanishing flow (EVF) has been subjected to intensive theoretical calculations using variety

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of equations of state as well as nucleon-nucleon cross-sections [1, 2, 3, 4, 5]. This also includes the mass dependence of EVF which have been reproduced successfully by various theoretical models [1, 3, 4].

Interestingly, most of these studies take symmetric or nearly symmetric reactions into account. Recently, FOPI group studied the flow for the asymmetric reaction of $^{40}Ca +^{197} Au$ [6]. They noted that the flow in the asymmetric collisions is a key observable for investigating the reaction dynamics. In contrast to symmetric collisions, where center-of-mass is one of the nucleus, this quantity is not known *a priori* in asymmetric nuclei experimentally. Later on, FOPI conducted experiment on ^{58}Ni and ^{208}Pb [6]. In other class of studies, the total mass of the system was kept fixed as 96 units whereas charge was varied [7]. Theoretically, recently Kaur and Kumar [8] conducted a complete study of the multifragmentation by varying the asymmetry of the colliding nuclei. Asymmetry parameter η is defined as $(A_T - A_p/A_T + A_P)$; where A_T and A_P are the masses of the target and projectile, respectively. All these attempts point towards a need for the systematic study of the disappearance of flow for asymmetric colliding nuclei. Further as noted, asymmetry of a reaction plays dramatic role in heavy-ion collisions [9]. This happens because excitation energy in symmetric colliding nuclei leads to larger compression while asymmetric reactions lack the compression since large part of the excitation energy is in the form of thermal energy.

Note that some isolated studies with asymmetric nuclei are already done in the literature where the reactions of $^{20}Ne+^{12}C$, $^{20}Ne+^{27}Al$, $^{20}Ne+^{63}Cu$, $^{58}Ni+^{12}C$, $^{64}Zn+^{27}Al$, $^{1}H+^{197}Au$, $^{12}C+^{197}Au$, $^{197}Au+^{12}C$, $^{40}Ar+^{207}Pb$ etc are taken into account [6, 10, 11]. Interestingly, none of the studies focus on the energy of vanishing flow in asymmetric colliding nuclei. To address this, we here present a systematic study of the EVF as a function of asymmetry of the colliding nuclei. While total mass of the reactions remain fixed, the asymmetry is varied by transferring the neutrons/protons from one nucleus to other. This study is performed with the quantum molecular dynamics (QMD) model which is explained briefly in section 2.

2. The quantum molecular dynamics model

The quantum molecular dynamics model (QMD) simulates the reaction on an event-by-event basis [2]. This is based on a molecular dynamics picture where nucleons interact via two- and three-body interactions. The explicit

two- and three-body interactions preserve the fluctuations and correlations which are important for N-body phenomenon such as multifragmentation [2].

In the model, the (successfully) initialized nuclei are boosted towards each other with proper center-of-mass velocity using relativistic kinematics. Here each nucleon α is represented by a Gaussian wave packet with a width of $\sqrt{L}$ centered around the mean position $\vec{r}_\alpha$(t) and mean momentum $\vec{p}_\alpha$(t) [2]:

$$\phi_\alpha(\vec{r},\vec{p},t) = \frac{1}{(2\pi L)^{3/4}} e^{\left[-\{\vec{r}-\vec{r}_\alpha(t)\}^2/4L\right]} e^{[i\vec{p}_\alpha(t)\cdot\vec{r}/\hbar]}. \tag{1}$$

The Wigner distribution of a system with $A_T + A_P$ nucleons is given by

$$f(\vec{r},\vec{p},t) = \sum_{\alpha=1}^{A_T+A_P} \frac{1}{(\pi\hbar)^3} e^{\left[-\{\vec{r}-\vec{r}_\alpha(t)\}^2/2L\right]} e^{\left[-\{\vec{p}-\vec{p}_\alpha(t)\}^2 2L/\hbar^2\right]}{}', \tag{2}$$

with L = 1.08 fm^2.

The center of each Gaussian (in the coordinate and momentum space) is chosen by the Monte Carlo procedure. The momentum of nucleons (in each nucleus) is chosen between zero and local Fermi momentum [$= \sqrt{2m_\alpha V_\alpha(\vec{r})}$; $V_\alpha(\vec{r})$ is the potential energy of nucleon α]. Naturally, one has to take care that the nuclei, thus generated, have right binding energy and proper root mean square radii.

The centroid of each wave packet is propagated using the classical equations of motion [2]:

$$\frac{d\vec{r}_\alpha}{dt} = \frac{dH}{d\vec{p}_\alpha}, \tag{3}$$

$$\frac{d\vec{p}_\alpha}{dt} = -\frac{dH}{d\vec{r}_\alpha}, \tag{4}$$

where the Hamiltonian is given by

$$H = \sum_\alpha \frac{\vec{p}_\alpha^{\,2}}{2m_\alpha} + V^{tot}. \tag{5}$$

Our total interaction potential V^{tot} reads as [2]

$$V^{tot} = V^{Loc} + V^{Yuk} + V^{Coul} + V^{MDI}, \tag{6}$$

with

$$V^{Loc} = t_1\delta(\vec{r}_\alpha - \vec{r}_\beta) + t_2\delta(\vec{r}_\alpha - \vec{r}_\beta)\delta(\vec{r}_\alpha - \vec{r}_\gamma), \tag{7}$$

$$V^{Yuk} = t_3 e^{-|\vec{r}_\alpha - \vec{r}_\beta|/m} / \left(|\vec{r}_\alpha - \vec{r}_\beta|/m\right), \tag{8}$$

with m = 1.5 fm and t_3 = -6.66 MeV.

The static (local) Skyrme interaction can further be parametrized as:

$$U^{Loc} = \alpha \left(\frac{\rho}{\rho_o}\right) + \beta \left(\frac{\rho}{\rho_o}\right)^{\gamma}. \tag{9}$$

Here α, β and γ are the parameters that define equation of state. The momentum dependent interaction is obtained by parameterizing the momentum dependence of the real part of the optical potential. The final form of the potential reads as

$$U^{MDI} \approx t_4 ln^2[t_5(\vec{p}_\alpha - \vec{p}_\beta)^2 + 1]\delta(\vec{r}_\alpha - \vec{r}_\beta). \tag{10}$$

Here t_4 = 1.57 MeV and t_5 = 5 $\times 10^{-4} MeV^{-2}$. A parameterized form of the local plus momentum dependent interaction (MDI) potential (at zero temperature) is given by

$$U = \alpha \left(\frac{\rho}{\rho_0}\right) + \beta \left(\frac{\rho}{\rho_0}\right) + \delta ln^2[\epsilon(\rho/\rho_0)^{2/3} + 1]\rho/\rho_0. \tag{11}$$

The parameters α, β, and γ in above Eq. (11) must be readjusted in the presence of momentum dependent interactions so as to reproduce the ground state properties of the nuclear matter. The set of parameters corresponding to different equations of state can be found in Ref. [2].

3. Results and discussion

For the present study, we simulated various reactions for 1000-5000 events in the incident energy range between 90 and 350 MeV/nucleon. In particular, we simulated the reactions of ${}^{17}_{8}O + {}^{23}_{11}Na$ (η = 0.1), ${}^{14}_{7}N + {}^{26}_{12}Mg$ (η = 0.3), ${}^{10}_{5}B + {}^{30}_{14}Si$ (η = 0.5), and ${}^{6}_{3}Li + {}^{34}_{16}S$ (η = 0.7) for total mass = 40, ${}^{36}_{18}Ar + {}^{44}_{20}Ca$ (η = 0.1), ${}^{28}_{14}Si + {}^{52}_{24}Cr$ (η = 0.3), ${}^{20}_{10}Ne + {}^{60}_{28}Ni$ (η = 0.5), and ${}^{10}_{5}B + {}^{70}_{32}Ge$ (η = 0.7) for total mass = 80, ${}^{70}_{32}Ge + {}^{90}_{40}Zr$ (η = 0.1), ${}^{54}_{26}Fe + {}^{106}_{48}Cd$ ($\eta = 0.3$), ${}^{40}_{20}Ca + {}^{120}_{52}Te$ ($\eta = 0.5$), and ${}^{24}_{12}Mg + {}^{136}_{58}Ce$ ($\eta = 0.7$) for total mass = 160, and ${}^{108}_{48}Cd + {}^{132}_{56}Ba$ ($\eta = 0.1$), ${}^{84}_{38}Sr + {}^{156}_{66}Dy$ ($\eta = 0.3$), ${}^{60}_{28}Ni + {}^{180}_{74}W$ (η = 0.5), and ${}^{36}_{18}Ar + {}^{204}_{82}Pb$ (η = 0.7) for total mass = 240. The present study is for semi-central collisions (i.e. $b/b_{max} = 0.25$). Note that in some cases, slight variation can be seen for charges. The charges are

chosen in a way so that colliding nuclei are stable nuclides. A soft equation of state with isotropic standard energy dependent cugnon cross-section (labeled as Soft^{iso}) and momentum dependent soft equation of state with standard energy dependent cugnon cross-section (labeled as SMD) are used in the present reactions.

The energy of vanishing flow (EVF) is calculated using the *directed transverse momentum* $< P_x^{dir} >$, which is defined as:

$$\langle P_x^{dir} \rangle = \frac{1}{A} \sum_i \text{sign}\{Y(i)\} \ \mathbf{p}_x(i), \tag{12}$$

where $Y(i)$ and $\mathbf{p}_x(i)$ are the rapidity distribution and transverse momentum of i^{th} particle, respectively.

In Fig. 1, we display the time evolution of the directed transverse flow $< P_x^{dir} >$, for various asymmetric reactions with system mass (sysmass) = 80 and 240 units. From the figure, it is clear that transverse flow saturates quite early during the interaction. A negative value during the initial stages signifies the dominance of the attractive interactions. One also sees well-known trends of transverse collective flow with increase in the incident energy. A clear transition from the negative flow to positive flow is also visible. From the figure, one can see that nearly symmetric reactions (η = 0.1 and 0.3) respond strongly to the change with incident energy compared to highly asymmetric reactions. One notices that for η=0.1 in the system mass 240, the change in the flow between 40 and 400 MeV/nucleon is 50 MeV, whereas for the same energy range, the flow varies between -2 MeV and +18 MeV (net 20 MeV) for η=0.7. The cause behind is that with the increase in the asymmetry of a reaction lesser binary collisions take place resulting in lesser density and therefore, less response occurs for variation in the collective flow. On the other hand, for nearly symmetric reactions, the binary collisions increase linearly with incident energy, therefore, huge difference can be seen.

It would be interesting to see how mass dependence character of the EVF behaves at a fixed asymmetry. In Fig. 2, we display the EVF verses combined system mass by keeping the asymmetry fixed. We notice that in all the cases, a perfect power law dependence (with power factor close to 1/3) can be seen for all asymmetries right from 0.1 to 0.7. All points lie on the line indicating that asymmetry plays a major role in the mass dependence. For a given mass e.g. system mass = 80 units, we see a variation of 50 MeV in EVF for η varying between 0.1 and 0.7. This also explains why in the earlier mass

dependence studies [1], though average behavior was a power law, individual EVFs were quite far from the average law [1, 3]. There this happened because no control was made for the asymmetry of a colliding pair. Had all reactions been analyzed on a fixed asymmetry, one could have obtain perfect match with average power law. Further, a careful comparison of Soft and SMD reveals that the difference between EVF with both methods narrows down for light nuclei with larger asymmetry. In another calculations at $b/b_{max} = 0.5$ (not shown here), one noticed that MDI gives smaller EVF for lighter nuclei whereas EVF with MDI is enhanced for heavier nuclei. The difference is clearly visible for larger asymmetries. The cause of this behavior lies in the fact that MDI are more attractive compared to soft static equation of state at low densities whereas they turn more repulsive at higher incident energies. Based on this information, it is clear that for lighter nuclei at large asymmetry (and therefore, low densities) MDI suppresses the EVF. These observations are more visible at large asymmetries.

In Fig. 3, we display EVF as a function of η for a fixed mass equal to 40, 80, 160, and 240 units. The results using the Soft^{iso} (top panel) and SMD (bottom panel) EoS are displayed for clarity. In agreement with all previous studies, EVF decreases with increase in the mass of the system with both Soft^{iso} and SMD EoS. This decrease has been attributed to the increasing role of Coulomb forces in heavier colliding nuclei. As discussed in earlier figures, a sizable influence can be seen towards EVF with variation of the asymmetry of a reaction. For lighter masses, the effect of the variation of η can result about 40 MeV change in the EVF. As noted in absolute terms, lighter nuclei are more affected compared to heavier ones. Overall, one sees that the effect of the asymmetry of a reaction is not at all negligible. It can have sizable effect which goes as power law with power factor close to 1/3.

In Fig. 4, we display the percentage difference ΔEVF(%) defined as $\Delta\text{EVF}(\%) = ((EVF^{\eta\neq 0}\text{-}EVF^{\eta=0})/EVF^{\eta=0})\times 100$. Very interestingly, we see that the effect of the asymmetry variation is almost uniform throughout the periodic table. The mean variation is 0.7%, 4%, 13%, and 39% with Soft^{iso} EoS and 0.3%, 2%, 6%, and 20% with MDI for $\eta = 0.1$, 0.3, 0.5, and 0.7, respectively. In other words, asymmetry of a reaction can play significant role in EVF and the deviation from the mean line can be eliminated if proper care is taken for the asymmetry of colliding nuclei.

In Fig. 5, we display the EVF as a function of the impact parameter for different asymmetries. A well known trend i.e. increase in the EVF with impact parameter can be seen. Further, as demonstrated by many

authors, the impact parameter variation is less effected in the presence of MDI. The striking result is that the effect of mass asymmetry variation is almost independent of the impact parameter.

4. Summary

Using the quantum molecular dynamics model, we presented a detailed study of the balance energy with reference to mass asymmetry. Almost independent of the system mass as well as impact parameter, a uniform effect of the mass asymmetry can be seen at the energy of vanishing flow. We find that for large asymmetries ($\eta = 0.7$), the effect of asymmetry can be 15% with MDI and in the absence it can be 40%. This also explains the deviation in the individual EVF from the mean values as reported earlier.

5. Acknowledgement

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Figure Captions

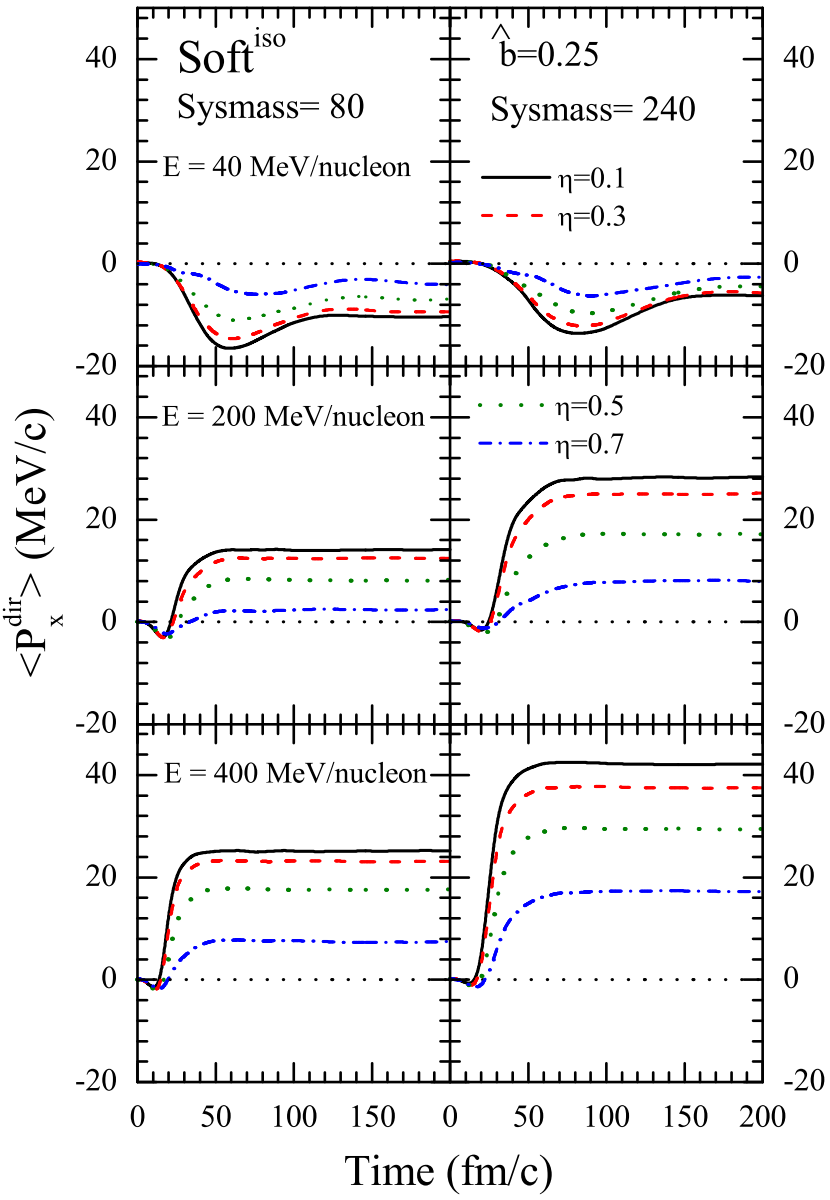
FIG. 1. (Color Online) The time evolution of the directed transverse flow $< P_x^{dir} >$ as a function of the reaction time using different mass asymmetries. The left panel is for the reaction leading to sysmass = 80 units whereas right panel indicates sysmass = 240 units. The results for different mass asymmetries $\eta = 0.1$, 0.3, 0.5, and 0.7 are represented, respectively, by the solid, dashed, dotted, and dashed-dotted lines.

FIG. 2. (Color Online) The EVF as a function of total mass (sysmass) of reacting partners. The upper panel is for Soft^{iso} EoS whereas lower panel is for SMD. The results for different asymmetries $\eta = 0.1$, 0.3, 0.5, and 0.7 are represented, respectively, by the solid squares, circles, triangles and inverted triangles. Lines are power law fit $\propto A^{\tau}$.

FIG. 3. (Color Online) The EVF as a function of asymmetry parameter η. The upper panel is for Soft^{iso} EoS whereas lower panel is for SMD. The results for the systems having total mass A of 40, 80, 160, and 240 are represented, respectively, by the open squares, circles, triangles and inverted triangles for the case of SMD whereas shaded area in the case of Soft^{iso} EoS represents the effect of charge variation leading to fixed value of charge and mass of the reaction. Lines are to guide the eye.

FIG. 4. (Color Online) The percentage difference ΔEVF(%) as a function of sysmass of reacting partners. The upper panel is for Soft^{iso} EoS whereas lower panel is for SMD. The results of the percentage difference for different asymmetries $\eta = 0.1$, 0.3, 0.5, and 0.7 are represented, respectively, by the solid squares, circles, triangles and inverted triangles. Horizontal lines represent the mean value of ΔEVF(%) for each η.

FIG. 5. (Color Online) The EVF as a function of reduced impact parameter for the sysmass 240. The upper panel is for Soft^{iso} EoS whereas lower panel is for SMD. The results of the percentage difference for different asymmetries $\eta = 0.1$, 0.3, 0.5, and 0.7 are represented, respectively, by the solid squares, circles, triangles and inverted triangles. Lines are to guide the eye.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E = 200 MeV/nucleon
E = 400 MeV/nucleon
b̂=0.25
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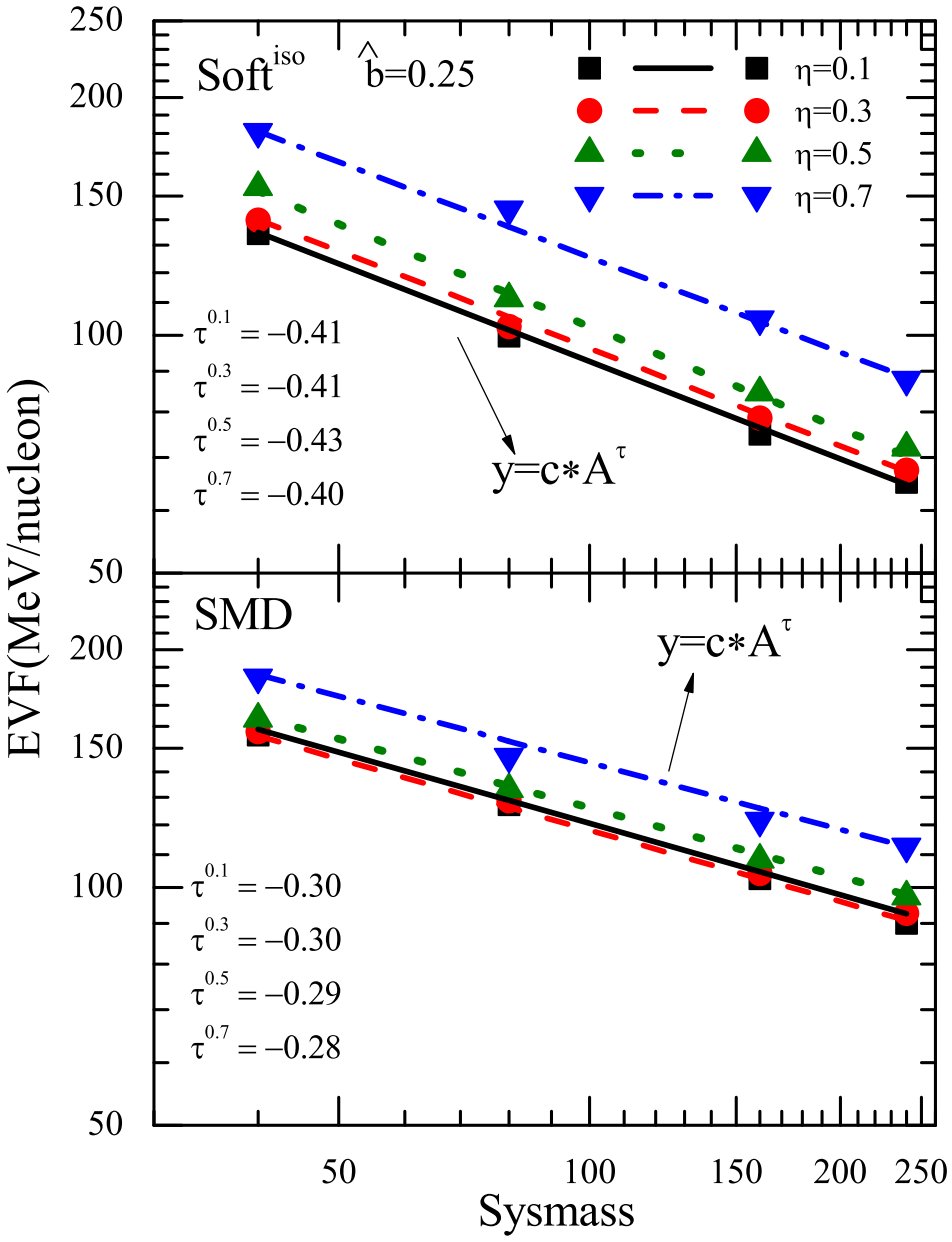
η=0.3
η=0.5
η=0.7
<P_x^dir> (MeV/c)
Time (fm/c)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τ^0.5 = –0.43
τ^0.7 = –0.40
y=c*A^τ
SMD
y=c*A^τ
τ^0.1 = –0.30
τ^0.3 = –0.30
τ^0.5 = –0.29
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Sysmass
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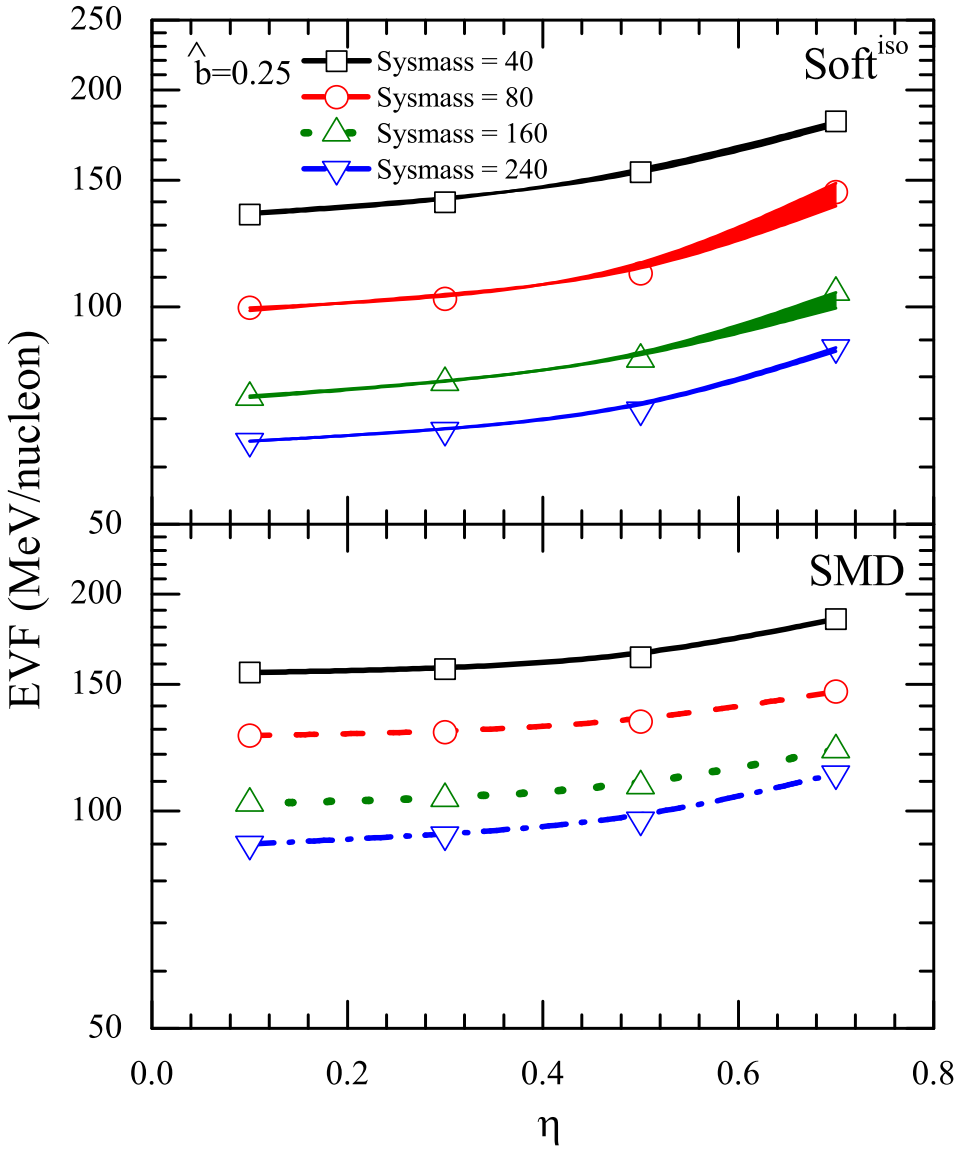
150
100
50
50
100
150
200
250


250
200
150
100
50
EVF (MeV/nucleon)
b̂=0.25
Sysmass = 40
Sysmass = 80
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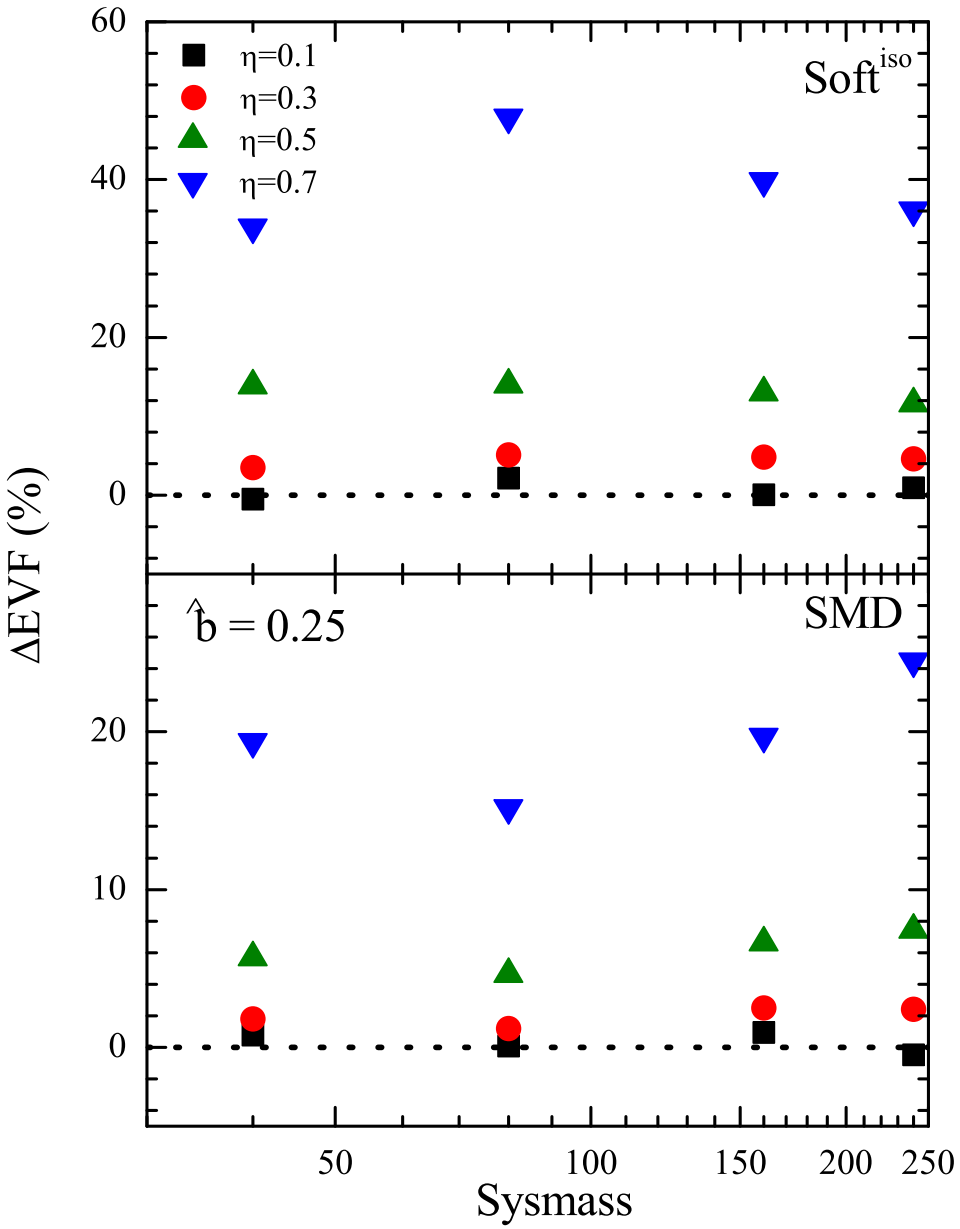
Sysmass = 240
Soft^iso
SMD
0.0
0.2
0.4
0.6
0.8
η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η=0.3
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ΔEVF (%)
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SMD
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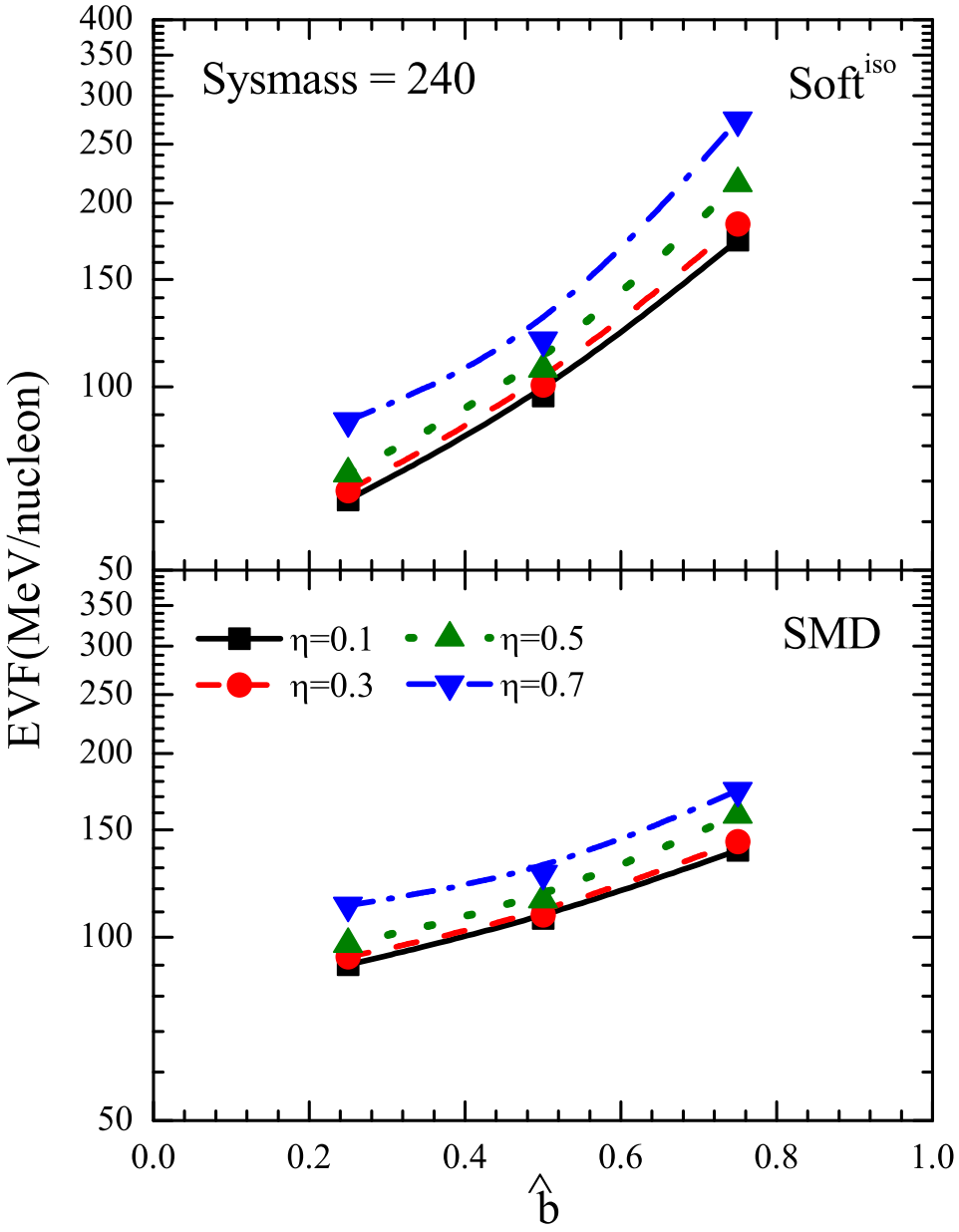
10
0
50
100
150
200
250
Sysmass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SMD
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η=0.7
EVF(MeV/nucleon)
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250
200
150
100
50
0.0
0.2
0.4
0.6
0.8
1.0
b̂